The Yennie–Frautschi–Suura exponentiation in leptonic W-boson decays

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Outline:

- **Introduction.**
- **Why is FSR in W production processes important?**
- **The YFS exponentiation in leptonic W decays.**
- **The Monte Carlo event genarator WINHAC.**
- **Numerical results.**
- **Conclusions and outlook.**

▷ W. Placzek & S. Jadach, hep-ph/0302065; $\longrightarrow$ http://cern.ch/placzek

Why to investigate W-boson production processes?

- To measure the Standard Model (SM) parameters, e.g. $M_W,\ \Gamma_W$

  ▷ PDG 2002: $\Delta M_W = 39$ MeV, $\Delta\Gamma_W = 42$ MeV,

  while: $\Delta M_Z = 2.1$ MeV, $\Delta\Gamma_Z = 2.3$ MeV.

- To test the SM, in particular its non-Abelian nature through triple and quartic gauge-boson couplings (TGC: WWV and QGC: WWV_1V_2).

- To get better constraints on the **Higgs mass**

  ▷ Indirectly from SM fits

  $\longrightarrow$ Requirements: $\Delta M_W \approx 0.7\times 10^{-2}\Delta m_t$ (for equal weights in χ^2 tests)

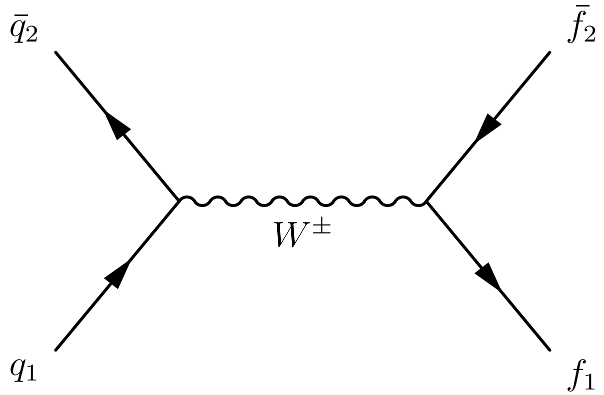
- To search for **"new physics"**, e.g. anomalous TGCs and QGC, etc.

- To measure parton distribition functions (PDF) and parton luminosity at LHC.

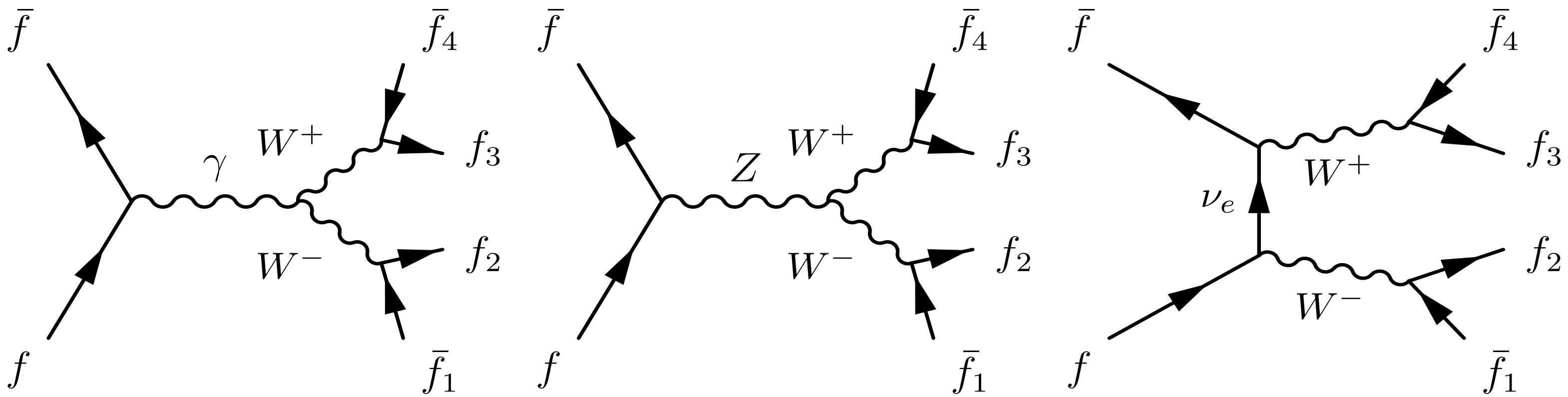
- Background for other processes, e.g. **Higgs boson** production.

Basic processes:

▷ **Single W:** $q_1 + \bar{q}_2 \longrightarrow W^{\pm} \longrightarrow f_1 + \bar{f}_2$



▷ **W-Pair:** $f + \bar{f} \longrightarrow W^-W^+ \longrightarrow f_1 + \bar{f}_2 + f_3 + \bar{f}_4 \qquad (f = e, q)$



Why is FSR in W production processes important?

W-boson mass and width measurement:

▷ Final-state radiation (FSR) distorts W-resonance line-shape

$\longrightarrow$ FSR reduces the effective W mass reconstructed from final fermion 4-momenta

Example:

Z-line shape from Z-pair production [Beenakker, Berends & Chapovsky, PL**B435** (1998) 233.]

(resummed – with soft-photon exponentiation)

▷ Z-peak distortions from FSR:

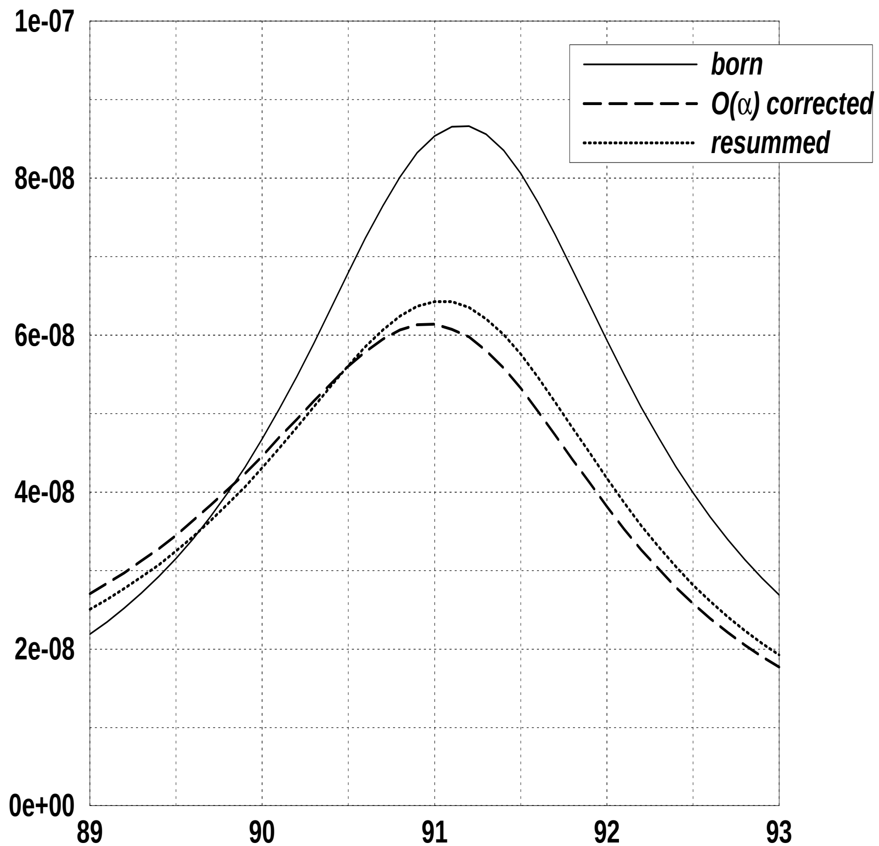
- $\mathcal{O}(\alpha)$:

$\Delta M_{peak} = -196$ MeV, $\kappa_{peak} = 0.70$

- resummed:

$\Delta M_{peak} = -111$ MeV, $\kappa_{peak} = 0.74$

$\longrightarrow$ W-peak distortions $\approx \frac{1}{2}$ of the above.

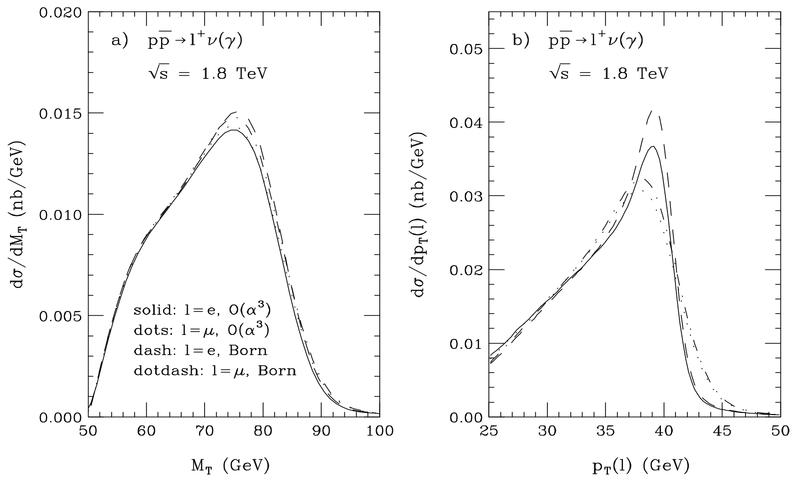


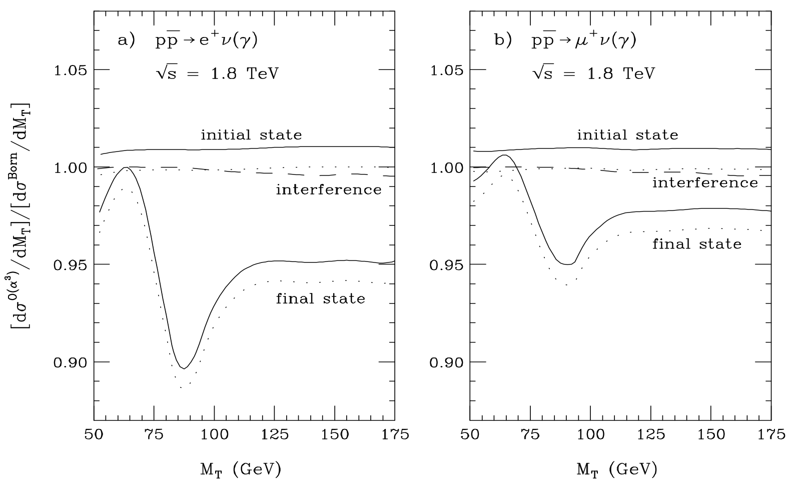
Z invariant mass from 'bare' e^+ and e^- 4-momenta

Why is FSR in W production processes important?

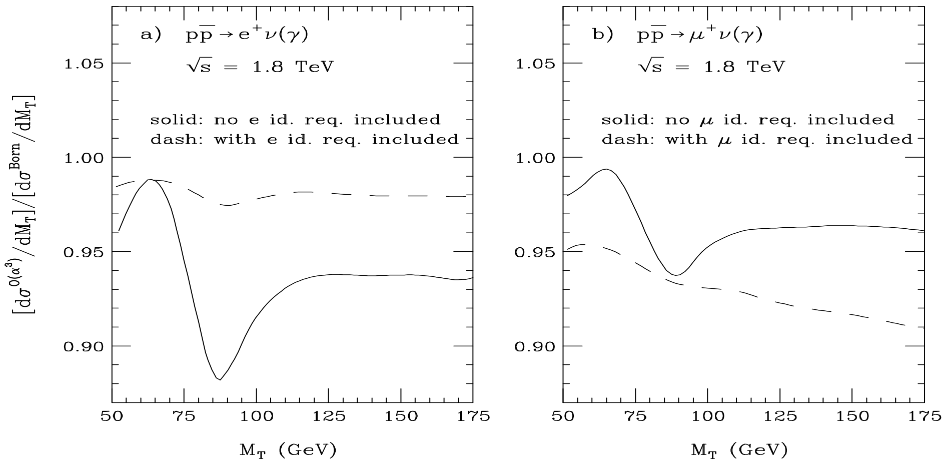
▷ Hadron colliders: M_W from W transverse mass M_T or final-lepton p_T

⟶ Tevatron: $\mathcal{O}(\alpha)$ radiative corrections [Baur, Keller & Wackeroth, PR**D59** (1998) 013002]





BARE vs. CALO acceptances

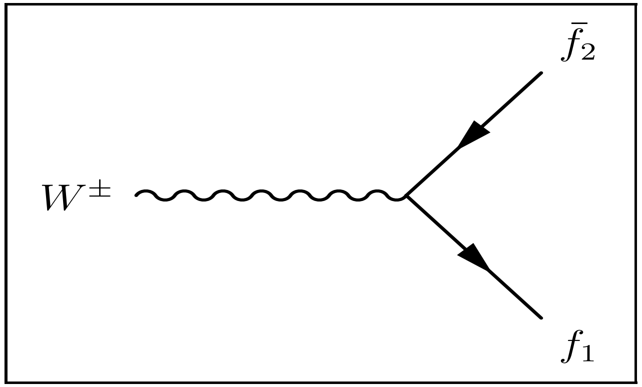


FSR effects are large and acceptance dependent ⇒ **MC event generator needed**

W-boson decay:

▷ Born-level process: $W^\pm \longrightarrow f_1 + \bar{f}_2,$

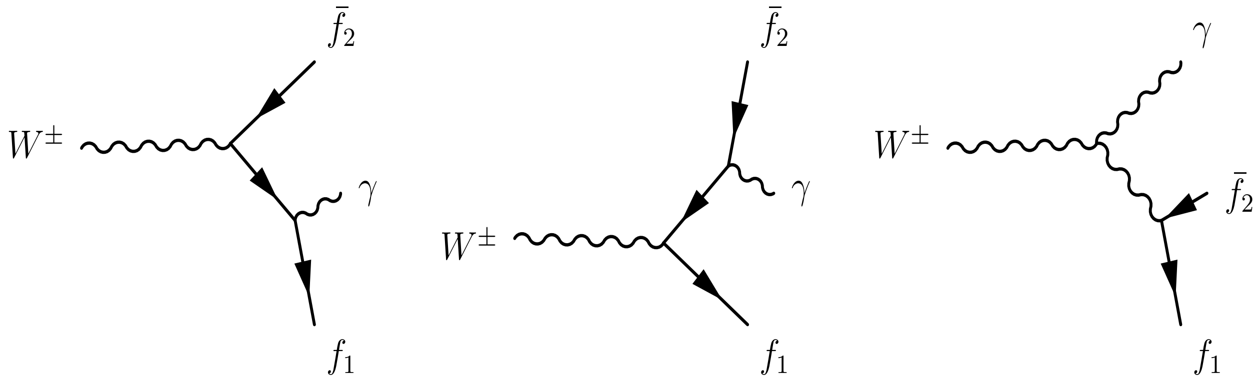
where $f_1, f_2 \in SU(2)_L$ doublets with $I_3^{f_1} = -I_3^{f_2}$.



Basic difference with Z-boson decay:

W is charged $\Rightarrow$ different electric charge flow and photon radiation from W-boson line

▷ Single photon radiation (in unitary gauge):



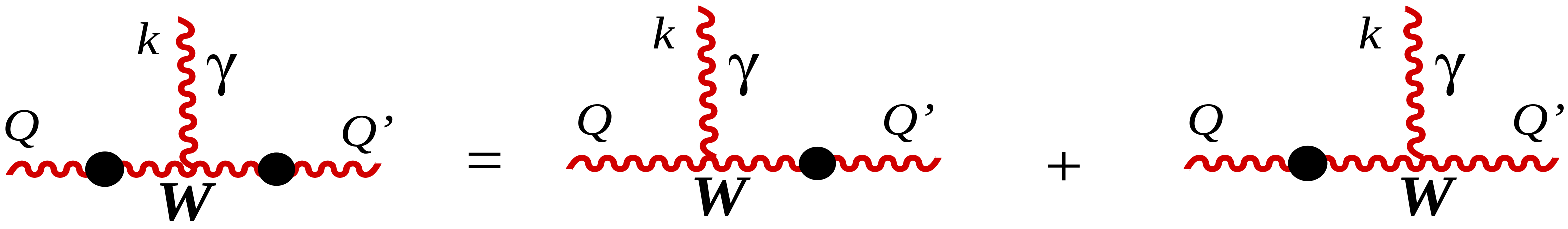
In actual processes:

▶ W-bosons in the intermediate state $\rightarrow$ W width must be included (preferably through the complex-pole definition)

The YFS exponentiation in leptonic W decays

▷ Photon emission from intermediate W-boson line

⟶ Partial-fraction decomposition of W propagators:



$$\frac{1}{Q^2 - M_W^2 + iM_W\Gamma_W} \times \frac{1}{Q'^2 - M_W^2 + iM_W\Gamma_W} =$$

$$\underbrace{\frac{1}{2kQ' + k^2}\,\frac{1}{Q'^2 - M_W^2 + iM_W\Gamma_W}}_{\leftarrow \text{ radiative production}} - \underbrace{\frac{1}{Q^2 - M_W^2 + iM_W\Gamma_W}\,\frac{1}{2kQ - k^2}}_{\text{radiative decay } \rightarrow}$$

⟹ Radiative corrections can be decomposed into:

a) radiative corrections to W production,

b) radiative corrections to W decay,

c) their interferences (non-factorizable).

▷ In resonance W production the non-factorizable corrections are negligible

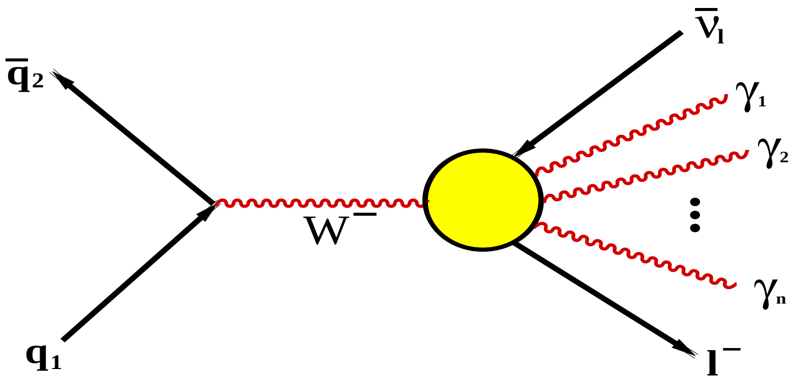
⟶ corrections to W-production and W-decay stages can be treated separately!

Here we investigate radiative corrections to leptonic W decays

▷ Single W-boson production in hadron collisions

- We consider the process:

$$q_1(p_1) + \bar{q}_2(p_2) \longrightarrow W^{\pm}(Q) \longrightarrow l(q_l) + \nu(q_\nu) + \gamma(k_1) + \ldots + \gamma(k_n), \quad (n = 0, 1, \ldots)$$



$\longrightarrow \mathcal{O}(\alpha)$ Yennie–Frautschi–Suura (YFS) exponentiated cross section:

$$\sigma^{tot}_{\rm YFS} = \sum_{n=0}^{\infty} \int \frac{d^3 q_l}{q_l^0} \frac{d^3 q_\nu}{q_\nu^0} \rho_n^{(1)}(p_1, p_2, q_1, q_2, k_1, \ldots, k_n),$$

where

$$\rho_n^{(1)} = e^{Y(Q,q_l;k_s)} \frac{1}{n!} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \tilde{S}(Q, q_l, k_i) \theta(k_i^0 - k_s)\, \delta^{(4)}\left(p_1 + p_2 - q_l - q_\nu - \sum_{i=1}^{n} k_i\right)$$
$$\times \left[\bar{\beta}_0^{(1)}(p_1, p_2, q_l, q_\nu) + \sum_{i=1}^{n} \frac{\bar{\beta}_1^{(1)}(p_1, p_2, q_l, q_\nu, k_i)}{\tilde{S}(Q, q_l, k_i)} \right].$$

▷ More details:

- **YFS FormFactor** – gauge-invariant resummation of IR contributions:

$$Y(Q,q_l;k_s) = \underbrace{2\alpha\Re B(Q,q_l;m_\gamma)}_{virtual\ photons} + \underbrace{2\alpha\tilde{B}(Q,q_l;m_\gamma,k_s)}_{real\ photons};$$

where

$$B(Q,q;m_\gamma) = \frac{i}{8\pi^3}\int\frac{d^4k}{k^2-m_\gamma^2+i\varepsilon}\left(\frac{2q-k}{k^2-2kq+i\varepsilon}-\frac{2Q-k}{k^2-2kQ+i\varepsilon}\right)^2,$$

$$\tilde{B}(Q,q;m_\gamma,k_s) = -\frac{1}{8\pi^2}\int_{k^0<k_s}\frac{d^3k}{k^0}\left(\frac{q}{kq}-\frac{Q}{kQ}\right)^2,$$

▷ Four-momentum transfer between charged particles:

$$t = (Q-q_l)^2 = \left(q_\nu + \sum_i k_i\right)^2 \geq 0$$

$\longrightarrow$ Different t domain than in production or scattering processes!

▷ We calculated this YFS formfactor in any Lorentz frame and for arbitrary particle masses $\longrightarrow$ numerically stable representations!

! Special care had to be taken for the cases of $t=0$ and W-rest frame (to avoid numerical instabilities).

▷ The virtual-photon IR function for $t > 0$ reads:

$$2\alpha\Re B(Q,q;m_\gamma) = \frac{\alpha}{\pi}\left\{[\,\nu A(Q,q) - 1\,]\,\ln\frac{m_\gamma^2}{Mm} + \frac{1}{2}A_1(Q,q) - \nu A_3(Q,q)\right\},$$

$$A(Q,q) = \frac{1}{\lambda}\ln\frac{\lambda+\nu}{Mm},$$

$$A_1(Q,q) = \frac{M^2-m^2}{t}\ln\frac{M}{m} - \frac{2\lambda^2}{t}A(Q,q) - 2,$$

$$\begin{aligned} A_3(Q,q) = A(Q,q)\,\ln\frac{2\lambda}{Mm} &+ \frac{1}{\lambda}\left[\frac{1}{4}\left(\ln\frac{\lambda+\nu}{M^2} + 2\ln\frac{\lambda-\nu+M^2}{t}\right)\ln\frac{\lambda+\nu}{M^2}\right. \\ &+ \frac{1}{4}\left(\ln\frac{\lambda+\nu}{m^2} - 2\ln\frac{\lambda+\nu-m^2}{m^2}\right)\ln\frac{\lambda+\nu}{m^2} \\ &\left.+ \frac{1}{2}\ln\eta\,\ln(1+\eta) - \frac{1}{2}\ln\zeta\,\ln(1+\zeta) + \Re\mathrm{Li}_2(-\eta) - \Re\mathrm{Li}_2(-\zeta)\right], \end{aligned}$$

$$\nu = Qq, \quad \lambda = \sqrt{(\nu - Mm)(\nu + Mm)}, \quad Q^2 = M^2,\; q^2 = m^2,\; M > m\,,$$

$$t = M^2 + m^2 - 2\nu, \qquad Mm \le \nu < \frac{1}{2}\left(M^2 + m^2\right),$$

$$\eta = \frac{m^2 t}{2\lambda(2\lambda + \nu - m^2)}, \qquad \zeta = \frac{\lambda+\nu}{m^2}\,\eta\,.$$

▷ The real-photon IR function for $t > 0$ reads:

$$\tilde{B}(Q,q;m_\gamma,k_s) = \frac{\alpha}{\pi}\left\{[\,\nu A(Q,q)-1\,]\,\ln\frac{4k_s^2}{m_\gamma^2} - \frac{M^2}{2}A_4(Q,Q) - \frac{m^2}{2}A_4(q,q) - \nu A_4(Q,q)\right\}$$

$$A_4(p,p) = \frac{1}{p^2\beta}\ln\frac{1-\beta}{1+\beta}, \qquad \beta = \frac{|\vec{p}|}{p^0},$$

$$A_4(Q,q) = \frac{1}{\kappa}\left\{\ln\left|\frac{V^2}{t}\right|\sum_{i=0}^{1}(-1)^{n+1}\,[\,X(z_i;y_1,y_4,y_2,y_3)+R(z_i)\,]\right\},$$

$$R(z) = Y_{14}(z)+Y_{21}(z)+Y_{32}(z)-Y_{34}(z)+\frac{1}{2}X(z;y_1,y_2,y_3,y_4)X(z;y_2,y_3,y_1,y_4)\,,$$

$$Y_{ij}(z) = 2Z_{ij}(z)+\frac{1}{2}\ln^2\left|\frac{z-y_i}{z-y_j}\right|, \qquad Z_{ij}(z) = \Re\mathrm{Li}_2\left(\frac{y_j-y_i}{z-y_i}\right),$$

$$X(z;a,b,c,d) = \ln\left|\frac{(z-a)(z-b)}{(z-c)(z-d)}\right|, \qquad z_0 = \frac{|\vec{q}|}{T}, \quad z_1 = \frac{|\vec{Q}|}{T}-1\,;$$

$$y_1 = -\frac{1}{2T}\left[T+\Omega-\frac{\omega\delta+\kappa}{t}V\right], \qquad y_2 = y_1-\frac{\kappa V}{tT},$$

$$y_3 = -\frac{1}{2T}\left[T-\Omega+\frac{\omega\delta+\kappa}{V}\right], \qquad y_4 = y_3+\frac{\kappa}{TV};$$

$$\kappa = \sqrt{(\omega^2-t)(\delta^2-t)}\,, \qquad \delta = M-m, \quad \omega = M+m\,,$$

$$T = \sqrt{\Delta^2-t}\,, \quad V = \Delta+T, \qquad \Delta = Q^0-q^0, \quad \Omega = Q^0+q^0\,.$$

- **The non-IR YFS functions:**

a) 0 real hard photons:

$$\bar{\beta}_0^{(1)}(p_1,p_2,q_l,q_\nu) = \bar{\beta}_0^{(0)}(p_1,p_2,q_l,q_\nu)\left[1+\delta^{(1)}(Q,q_l,q_\nu)\right]$$

where: $\bar{\beta}_0^{(0)} = \frac{1}{8s\,(2\pi)^2}\frac{1}{12}\sum\left|\mathcal{M}^{(0)}\right|^2$ $\longleftarrow$ Born-like contribution

▶ $\mathcal{O}(\alpha)$ electroweak virtual corrections:

$$\delta^{(1)}(Q,q_l,q_\nu) = \delta^v_{\rm EW}(Q,q_l,q_\nu;m_\gamma) - 2\alpha\Re B(Q,q_l;m_\gamma)$$

▶ In the current version only QED-like corrections included:
[based on: Marciano & Sirlin, PR **D8** (1973) 3612]

$$\delta^{(1)}_{\rm QED}(Q,q_l) = \frac{\alpha}{\pi}\left(\ln\frac{M}{m_l}+\frac{1}{2}\right)$$

b) 1 real hard photon:

$$\bar{\beta}_1^{(1)}(p_1,p_2,q_l,q_\nu,k) = \frac{1}{16s\,(2\pi)^5}\frac{1}{12}\sum\left|\mathcal{M}^{(1)}\right|^2 - \tilde{S}(Q,q_l,k)\bar{\beta}_0^{(0)}(p_1,p_2,q_l,q_\nu),$$

where: $\tilde{S}(Q,q_l,k) = -\frac{\alpha}{4\pi^2}\left(\frac{Q}{kQ}-\frac{q_l}{kq_l}\right)^2$ $\longleftarrow$ soft-photon factor

▷ Matrix elements:

$$\mathcal{M}^{(0)}(\sigma_1,\sigma_2;\tau_1,\tau_2) = \frac{1}{Q^2 - M_W^2 + iM_W\Gamma_W} \sum_{\lambda} \mathcal{M}_P^{(0)}(\sigma_1,\sigma_2;\lambda)\,\mathcal{M}_D^{(0)}(\lambda;\tau_1,\tau_2)$$

$$\mathcal{M}^{(1)}(\sigma_1,\sigma_2;\tau_1,\tau_2,\kappa) = \frac{1}{Q^2 - M_W^2 + iM_W\Gamma_W} \sum_{\lambda} \mathcal{M}_P^{(0)}(\sigma_1,\sigma_2;\lambda)\,\mathcal{M}_D^{(1)}(\lambda;\tau_1,\tau_2,\kappa)$$

► Spin amplitudes in Weyl-spinor representation [cf. Hagiwara & Zeppenfeld, NP **B274** (1986) 1]:

a) Born-level W production:

$$\mathcal{M}_P^{(0)}(\sigma_1,\sigma_2;\lambda) = -\frac{ieV_{f_1f_2}}{\sqrt{2}s_W}\,\omega_{-\sigma_1}(p_1)\,\omega_{\sigma_2}(p_2)\,\sigma_2\, S\left(p_2,\epsilon_W^*(Q,\lambda),p_1\right)^{-}_{-\sigma_2,\sigma_1}$$

b) Born-level W decay:

$$\mathcal{M}_D^{(0)}(\lambda;\tau_1,\tau_2) = -\frac{ieCV_{f_1f_2}}{\sqrt{2}s_W}\,\omega_{-\tau_1}(q_1)\,\omega_{\tau_2}(q_2)\,\tau_2\, S\left(q_1,\epsilon_W(Q,\lambda),q_2\right)^{-}_{\tau_1,-\tau_2}$$

c) W decay with single real-photon radiation:

$$\begin{aligned}
\mathcal{M}_D^{(1)}(\lambda;\tau_1,\tau_2,\kappa) &= -\frac{ie^2CV_{f_1f_2}}{\sqrt{2}s_W}\,\omega_{-\tau_1}(q_1)\,\omega_{\tau_2}(q_2)\,\tau_2 \\
&\times\left\{\left(\frac{Q_{f_1}\,q_1\cdot\epsilon_\gamma^*}{k\cdot q_1} - \frac{Q_{f_2}\,q_2\cdot\epsilon_\gamma^*}{k\cdot q_2} - \frac{Q_W\,Q\cdot\epsilon_\gamma^*}{k\cdot Q}\right) S\left(q_1,\epsilon_W(Q,\lambda),q_2\right)^{-}_{\tau_1,-\tau_2}\right. \\
&+\frac{Q_{f_1}}{2\,k\cdot q_1} S\left(q_1,\epsilon_\gamma^*(k,\kappa),k,\epsilon_W(Q,\lambda),q_2\right)^{-}_{\tau_1,-\tau_2} - \frac{Q_{f_2}}{2\,k\cdot q_2} S\left(q_1,\epsilon_W(Q,\lambda),k,\epsilon_\gamma^*(k,\kappa),q_2\right)^{-}_{\tau_1,-\tau_2} \\
&\left.-\frac{Q_W\,k\cdot\epsilon_W}{2\,k\cdot Q} S\left(q_1,\epsilon_\gamma^*(k,\kappa),q_2\right)^{-}_{\tau_1,-\tau_2} + \frac{Q_W\,\epsilon_W\cdot\epsilon_\gamma^*}{2\,k\cdot Q} S\left(q_1,k,q_2\right)^{-}_{\tau_1,-\tau_2}\right\}
\end{aligned}$$

$\longrightarrow$ Spin amplitudes evaluated numerically for arbitrary fermion masses!

- **Monte Carlo algorithm for multiphoton radiation:**

▷ Lorentz frame choice for low-level MC generation

- ▶ Our previous MC generators:
  - a) ISR in annihilation processes ⟶ initial-beams CMS
  - b) FSR in neutral boson decays ⟶ final-state fermion-pair CMS
  - c) Bhabha scattering ⟶ electron/positron Breit frames
- ▶ W-boson decay ⟶ W-rest frame (seems most natural)

▷ Construction of MC algorithm:

⟶ Step-by-step simplification of the YFS formula for the cross section, compensated with appropriate MC weights – until Poissonian distribution is reached.

⟶ Generation of random variables and evaluation of compensating weights – in the opposite way to the above simplification process.

⟶ Construction of a MC event in terms of particle flavours and 4-momenta.

▷ MC event generator **WINHAC version 1.11**: ⟶ http://cern.ch/placzek

⟶ Full-hadron level (Tevatron/LHC): quark x and Q^2 generated with the help of self-adapting MC sampler **Foam** according to PDFs from **PDFLIB** package.

• Basic tests at the parton level

▷ Test of spin amplitudes and MC algorithm:

$\longrightarrow$ To reproduce Born-level and $\mathcal{O}(\alpha)$ results from the YFS exponentiation

a) Born cross section:

$$\sigma_0^{\rm WH} = \int \frac{d^3 q_l}{q_l^0} \frac{d^3 q_\nu}{q_\nu^0}\, \rho_0^{(0)}\, e^{-Y}$$

$$\sigma_0^{\rm An} = \frac{\alpha^2 \pi |V_{ij}|^2}{36 s_W^4} \times \frac{s}{(s-M_W^2)^2 + M_W^2 \Gamma_W^2}$$

Calculation	σ_0^{tot} [nb]		
	e	μ	τ
Analytical	8.8872	8.8872	8.8872
WINHAC	$8.8869\,(2)$	$8.8873\,(2)$	$8.8808\,(2)$

b) $\mathcal{O}(\alpha)$ corrected cross section:

$$\sigma_1^{\rm WH} = \int \frac{d^3 q_l}{q_l^0} \frac{d^3 q_\nu}{q_\nu^0}\, \delta^{(4)}(p_1+p_2-q_l-q_\nu)\, \bar\beta_0^{(0)} \left[\, 1 + \delta^{(1)}_{\rm QED} + Y \,\right]$$
$$+ \int \frac{d^3 q_l}{q_l^0} \frac{d^3 q_\nu}{q_\nu^0} \frac{d^3 k}{k^0}\, \delta^{(4)}(p_1+p_2-q_l-q_\nu-k) \left[\, \bar\beta_1^{(1)} + \tilde S \bar\beta_0^{(0)} \,\right] \theta(k^0-k_s)\,,$$

$$\delta_1^{\rm An} = \frac{\alpha}{\pi} \left(\frac{77}{24} - \frac{\pi^2}{3} \right) \approx 1.89 \times 10^{-4}$$

Calculation	$\delta_1 = \sigma_1^{tot}/\sigma_0^{tot} - 1$		
	e	μ	τ
WINHAC	$-1.5\,(3) \times 10^{-4}$	$-2.2\,(3) \times 10^{-4}$	$-0.3\,(2) \times 10^{-4}$

▶ Hard photon spectrum at $\mathcal{O}(\alpha)$

$$\delta_1^h(k_0) = \frac{1}{\sigma_1^{tot}} \int_{E_0} dE_\gamma \, \frac{d\sigma_1}{E_\gamma} \times 100\%$$

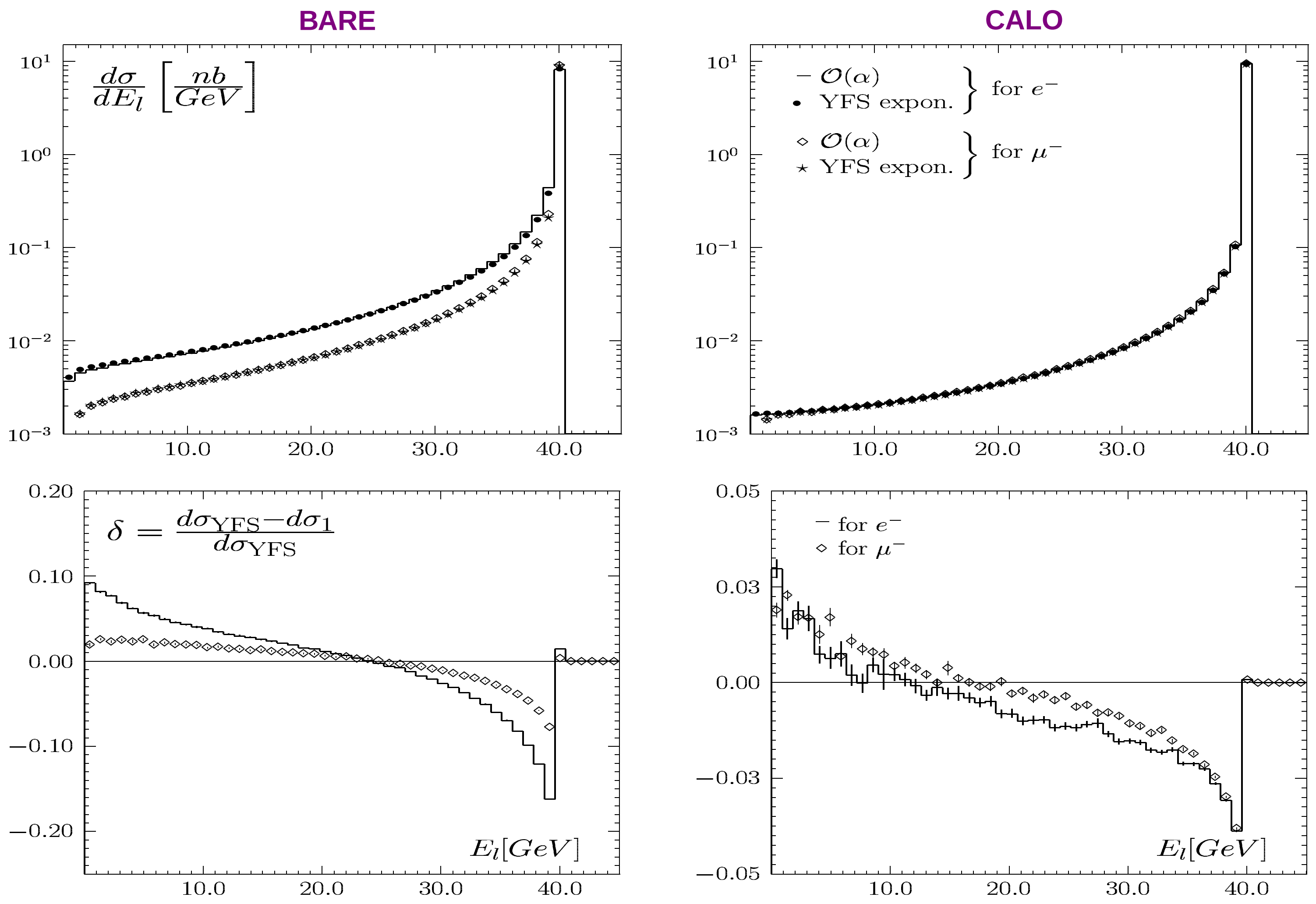
$E_0 = k_0 \times E_{\rm CM}/2, \qquad E_{\rm CM} = 90$ GeV

B&K: Berends & Kleiss, ZP **C27** (1985) 365

k_0	e		μ	
	WINHAC	B&K	WINHAC	B&K
0.01	19.69 (3)	19.7	10.11 (2)	10.1
0.05	11.61 (2)	11.6	5.92 (1)	5.9
0.10	8.31 (2)	8.3	4.22 (1)	4.2
0.15	6.47 (2)	6.5	3.27 (1)	3.3
0.20	5.23 (1)	5.2	2.63 (1)	2.6
0.30	3.61 (1)	3.6	1.80 (1)	1.8
0.40	2.57 (1)	2.6	1.27 (1)	1.3
0.50	1.84 (1)	1.8	0.91 (1)	0.9
0.60	1.29 (1)	1.3	0.63 (1)	0.6
0.70	0.86 (1)	0.9	0.42 (1)	0.4
0.80	0.52 (1)	0.5	0.25 (1)	0.2
0.90	0.24 (1)	0.2	0.11 (1)	0.1

WINHAC reproduces very well Born and $\mathcal{O}(\alpha)$ calculations!

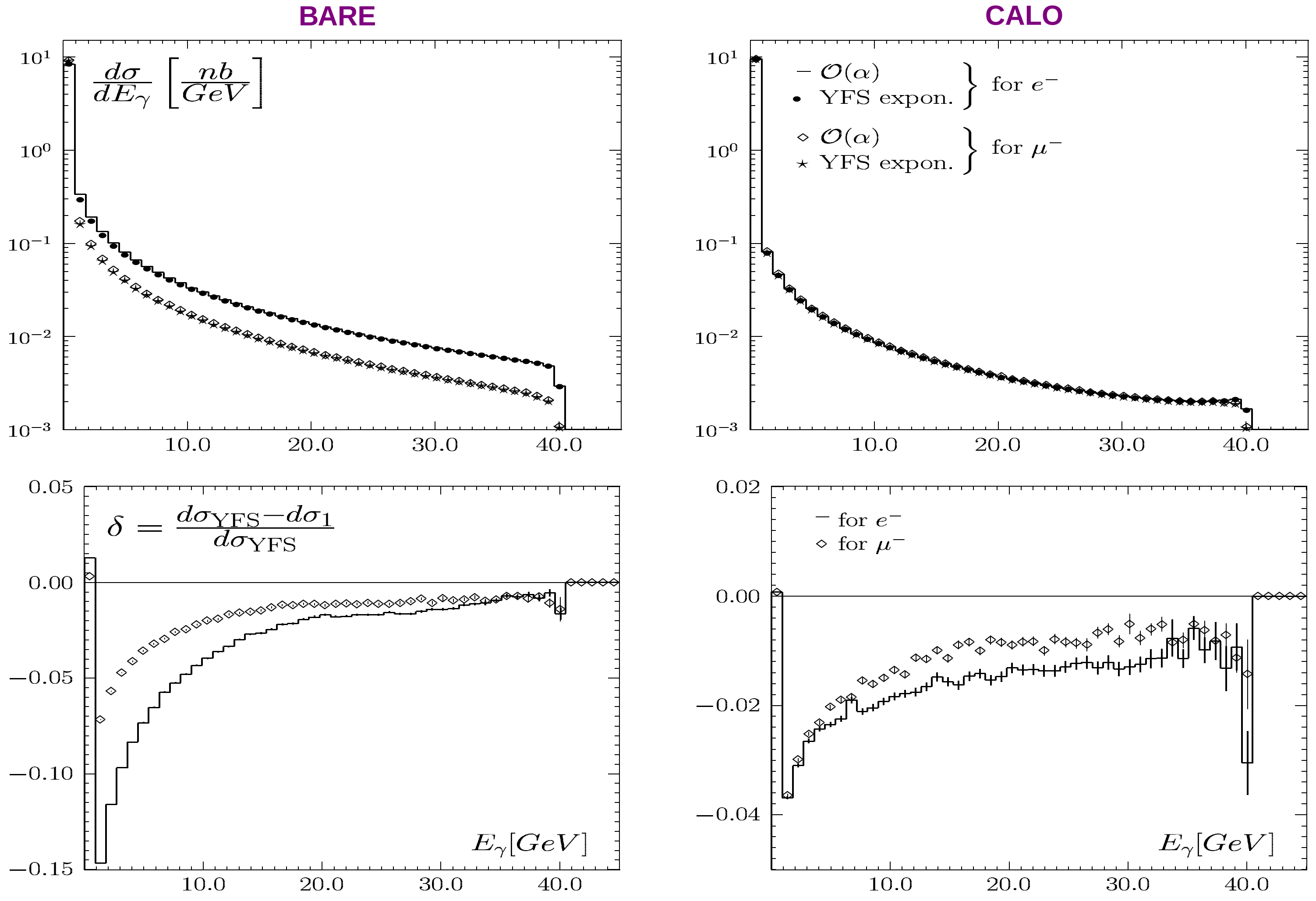
- **Parton level distributions – higher-order FSR effects:**
  - ▷ Charged-lepton energy E_l



CALO: Photons recombined with charged lepton if $\angle(l,\gamma) \leq 5^\circ$.

Numerical results

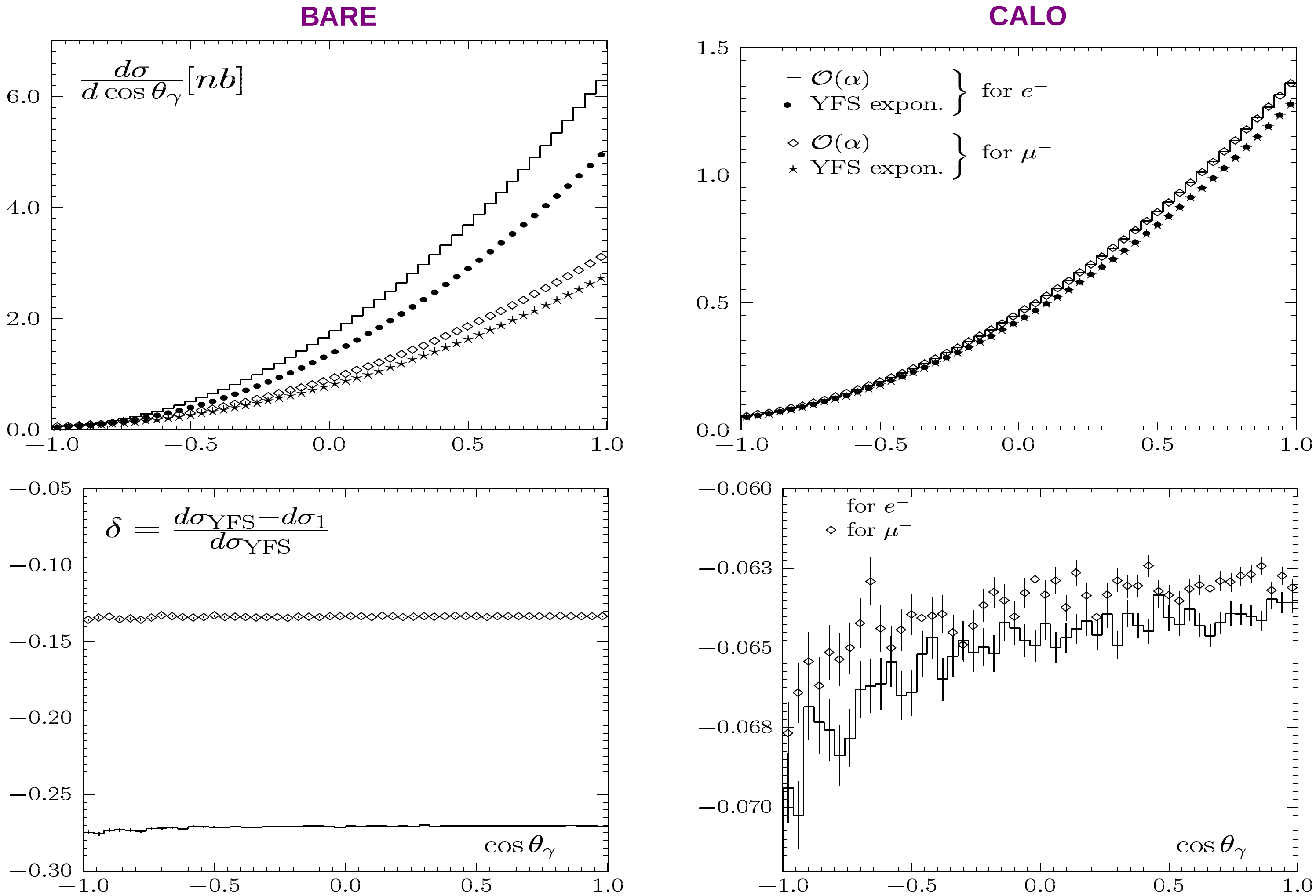
▷ Hardest-photon energy E_γ:



CALO: Photons recombined with charged lepton if $\angle(l,\gamma) \leq 5^\circ$.

Numerical results

▷ *Cosine* of hardest-photon polar angle $\cos\theta_\gamma$:



CALO: Photons recombined with charged lepton if $\angle(l,\gamma) \leq 5^\circ$.

Higher-order QED FSR effects are sizeable and acceptance dependent!

Conclusions

- We calculated multiphoton radiation in leptonic W-boson decays in the YFS exclusive exponentiation scheme.
- Appropriate Monte Carlo algorithm has been constructed.
- The above have been implemented in the MC event generator **WINHAC** for single W-boson production in hadron collisions (Tevatron/LHC).
  $\longrightarrow$ Acceptance efficiency: $\approx 50\%$
  $\longrightarrow$ CPU time: $\approx 10,000$ events per second on Pentium IV, 2.4 GHz.
- We also constructed an interface called **WINDEC** for multiphoton leptonic W-decay generator and any external MC program providing single W production in hadron collisions.

... and Outlook

- Inclusion of full $\mathcal{O}(\alpha)$ EW corrections to single W production.
  $\longrightarrow$ In collaboration Dubna group of Dima Bardin.
- Inclusion of QCD effects in W production.